INTERACTION OF THE RADIATION FROM A FLASH TUBE WITH THE COOLING LIQUID

M. I. Tribel'skii

UDC 536.3

An analytical solution is found for the nonlinear hydrodynamic problem describing the reaction of the cooling liquid which partially absorbs radiation from a flash tube, in the case of a single flash of the latter. A method of reducing the pressure in the liquid, in order to increase the limiting load of the flash tube, is described.

Due to the limiting load and life of flash tubes operating in a cooling liquid which absorbs the part of their radiation which is harmful for the active element of a laser, in comparison with the same parameters for flash tubes operating in a medium which does not possess filtering properties, the possibility arises of calculating the temperature and pressure developed in the liquid by the action of the radiation from the flash tube. A calculation of this nature was carried out within the framework of linear acoustics in [1]. In this present paper, the results are obtained in linear approximation with respect to density and in nonlinear approximation with respect to temperature of the liquid, which expands significantly their range of applicability in comparison with [1].

The liquid fills the space between two coaxial cylinders — the flash tube vessel and the reflector. We shall assume that the flash tube and reflector have an infinite length along the axis of axial symmetry. Then, in view of the symmetry of the problem, all quantities will depend only on the distance to the axis of the cylinders and the time (r and t, respectively).

We shall assume that the relation

$$\tau \gg \frac{R_2 - R_1}{u} \tag{1}$$

is satisfied. Since the pressure in the liquid is smoothed with the velocity of sound, condition (1) shows that the pressure in the laser is a function only of time.

As will be seen later, without violating generality, it can be assumed that the coolant is defined independently of the frequency by the absorption coefficient over a certain range of frequencies of the incident radiation, and in the remaining spectral region it is absolutely transparent.

Then the processes taking place in the liquid are described by the following equations:

$$\begin{cases} \rho \left\{ c_p \dfrac{\partial T}{\partial t} - \dfrac{\alpha T}{\rho} \dfrac{dp}{dt} \right\} = -(\vec{\nabla}, \mathbf{q}), \\ \mathbf{q} = -\varkappa \vec{\nabla} T + \mathbf{S}, \\ (\vec{\nabla}, \mathbf{S}) = -K S, \\ p(t) = p(\rho, T), \end{cases} \tag{2}$$

where $p(\rho, T)$ is the equation of state (assumed to be known).

The first equation of system (2) is a consequence of the law of conservation of energy $-(\vec{\nabla}, \mathbf{q})dt = \rho d\varepsilon - (p/\rho^2)d\rho$, and the hydrodynamic flows in q can be neglected in view of condition (1).

System (2), together with the conditions $T(0, r) = T_0$, $\rho(0, r) = \rho_0$(t = 0 corresponds to the start of

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 28, No. 2, pp. 263-267, February, 1975. Original article submitted May 29, 1974.

the flash), $S(t, R_1) = w(t)$, the conditions of thermal exchange at the boundary, and the condition of conservation of mass $\int_V \rho dV = \text{const}$ defines completely the state of the liquid.*

Because the temperature of the liquid is much less than the value at which thermal ionization of the molecules becomes significant, the coefficient K can be assumed to be independent of the temperature.

But since the principal mechanism of radiation absorption is the absorption of a quantum by an individual molecule, we obtain that $K = k\rho$, where k is constant for a given coolant quantity.

We shall suppose that $\rho = \rho_0 + \rho_1$; $|\rho_1| \ll \rho_0$, and we shall take into account only the first nonvanishing terms in the expansion with respect to ρ_1.

With these assumptions, the third equation of system (2) is integrated at once and we obtain

$$S = w(t) \frac{R_1}{r} \exp\{-k\rho_0 (r - R_1)\} \equiv w(t) g(r). \tag{3}$$

Because $K \simeq k\rho$, and the liquid expands when heated, the expression for S is stable relative to small changes of ρ, and therefore, the substitution of $k\rho$ by $k\rho_0$ cannot introduce a large error.†

Since the thermal conductivity of the liquid is small, during the entire flash $|\varkappa \vec{\nabla} T| \ll S$, i.e., $\mathbf{q} \simeq S$ (all numerical estimates will be made at the end of the paper).

The equation of state of the liquid, applicable for its change of temperature and density over a wide range, is considered in [2]. However, for the analysis conducted below, it is not required to know the specific form of the function $p(\rho, T)$.

With the stated accuracy of the quantity c_p, α and β are functions only of temperature, i.e., $c_p(\rho, T) \simeq c_p(\rho_0, T) \equiv c_p(T)$, etc.

Then, by using the condition for conservation of mass, it is easy to obtain that

$$\frac{dp}{dt} \int_V \beta(T)\, dV - \int_V \alpha(T) \frac{\partial T}{\partial t}\, dV = 0. \tag{4}$$

Substituting dp/dt, found from Eq. (4) in the first equation of system (2), we obtain, finally,

$$c_p(T) \frac{\partial T}{\partial t} \int_V \beta(T)\, dV - \frac{T\alpha(T)}{\rho_0} \int_V \alpha(T) \frac{\partial T}{\partial t} dV = kg(r) w(t) \int_V \beta(T)\, dV. \tag{5}$$

Before solving Eq. (5), we note that the main part of the radiation is absorbed in a thin layer $\sim(k\rho_0)^{-1}$ close to the flash tube. Therefore, the temperature in this layer increases considerably. In the remaining volume, the temperature changes only slightly because of the almost adiabatic compression of the coolant.

Moreover, by using the thermodynamic identities, it can be shown that

$$\frac{T\alpha^2}{\rho c_p \beta} = 1 - \gamma^{-1} < 1.$$

From all that has been said, it is clear that the second term in the left-hand side of Eq. (5) in the region of absorption is much less than the first term. Discarding it, we obtain the equation defining T in the region $r - R_1 \lesssim (k\rho_0)^{-1}$.

$$\int_{T_0}^{T} c_p(\theta)\, d\theta = kg(r) \int_0^t w(\xi)\, d\xi. \tag{6}$$

In the region $r - R_1 \gg (k\rho_0)^{-1}$, the inequality $T - T_0 \ll T_0$ is satisfied, and therefore, we can put with the stated accuracy that $c_p(T) \simeq c_p(T_0) \equiv c_{p0}$; $\alpha(T) \simeq \alpha(T_0) \equiv \alpha_0$; and $\beta(T) \simeq \beta(T_0) \equiv \beta_0$. Denoting the volume of the absorption layer by V_c; $1/\beta_0 \int_{V_c} \beta(T) dV$ by f(t) and $1/\beta_0 \int_{V_c} \alpha(T)(\partial T/\partial t) dV$ by F(t) [T(**r**, t),

*In view of the translational symmetry along the axis of the cylinders, here and in future we shall understand by V, the volume arriving per unit length of the axis of axial symmetry.

†If, over the range of frequencies being filtered, the function $k(\omega)$ is significant (ω is the frequency of the radiation), then the expression of the type of Eq. (3) is valid only for the differential S. In order to obtain the total energy flow density, it must be integrated with respect to frequency. In the remainder of the scheme of calculation, it remains without change.

which is the solution of Eq. (6), is substituted in the integrals], we obtain that Eq. (5) in the region $r - R_1 \gg (k\rho_0)^{-1}$, in linear approximation with respect to $T - T_0$, has the form*

$$\beta_0 c_{p0} \frac{\partial T}{\partial t} [V + f(t)] - \frac{\alpha_0 \beta_0}{\rho_0} TF(t) - \frac{T_0 \alpha_0^2}{\rho_0} \int\limits_{V-V_c} \frac{\partial T}{\partial t} dV = 0. \tag{7}$$

Integrating Eq. (7) over the volume $V - V_c$, we obtain the differential equation which defines $\int\limits_{V-V_c} TdV$. Expanding its solution in Taylor series and limiting it to the first nontrivial term, we obtain that

$$\int\limits_{V-V_c} TdV \simeq T_0 V \left\{1 + \int\limits_0^t \frac{\alpha_0 \beta_0 F(\xi)\, d\xi}{\rho_0 \beta_0 c_{p0} [V + f(\xi)] - T_0 V \alpha_0^2}\right\}.$$

Returning to Eq. (4), we obtain, finally,

$$p = p_0 + \int\limits_0^t d\xi \frac{F(\xi)}{V + f(\xi)} \left\{1 + \frac{\alpha_0^2 T_0 V}{\rho_0 \beta_0 c_{p0} [V + f(\xi)] - T_0 V \alpha_0^2}\right\}. \tag{8}$$

Expression (8) can be simplified somewhat, if we take into account that, generally,

$$\frac{T_0 \alpha_0^2}{\rho_0 \beta_0 c_{p0}} \ll 1 \text{ and } \max_t \{f(t)\} \ll V$$

[the latter is because $\beta(T)$ varies within finite limits in a small region V_c]. In this case

$$p \simeq p_0 + \frac{\gamma_0}{V} \int\limits_0^t F(\xi)\, d\xi, \tag{9}$$

Here $\gamma_0 \equiv \gamma(T_0)$. Knowing the function p(t), it is not difficult to determine T(r, t) and $\rho_1(r, t)$. We have

$$T = T_1 + T_0 \frac{\alpha_0 (p - p_0)}{\rho_0 c_{p0}}. \tag{10}$$

Here T_1 is the solution of Eq. (6),

$$\rho_1 = \beta(T) \rho_0 [p(t) - p(T, \rho_0)].$$

As F(t) is independent of V, then the excess pressure originating in the coolant is inversely proportional to the volume of liquid.

If c_p and α depend weakly on T, then it can be assumed that $c_p \simeq c_{p0}$ and $\alpha \simeq \alpha_0$ also in the region V_c. Then expression (9) gives a result which coincides with that obtained in [1] for the corresponding case.

In the calculation given, boiling of the coolant has not been taken into account. This is valid if transition to the final state is achieved by a path which does not intersect the phase equilibrium curve. If the liquid starts to boil and boiling takes place far from the critical point and as $\rho_0 \gg \rho_n$, but the total volume is fixed, then boiling must lead to a significant increase of pressure. The pressure can be determined from the formula

$$\Delta p \sim u^2 \frac{n(\rho_0 - \rho_n)}{1 - n},$$

to an order of magnitude of the pressure rise, where n is the ratio of the volume occupied by the gas phase to the total volume. This type of situation frequently is achieved in an experiment [3].

A quantitative computation of the effect of boiling on the distribution of pressure, density, and temperature in the liquid requires separate consideration and falls outside the scope of this present paper.

It should be noted that formulas (8)-(10) remain valid, even if $|\rho_1| \sim \rho_0$ in the region V_c. Actually, in this region the limitation of $|\rho_1| \ll \rho_0$ for a given flash energy is the limitation on the quantity K, while for the magnitude of the pressure only the total quantity of absorbed energy is important, and therefore, in zero approximation p generally is independent of K. In its turn, Eq. (6), which defines T in the region V_c,

*From the existence of the region $r - R_1 \gg (k\rho_0)^{-1}$, it follows that $V \gg V_c$. This inequality was used in the derivation of Eq. (7).

shows that in the stated region the liquid can be assumed to be heated up isobarically. However, the latter is conditioned by the properties of the liquid and not by the smallness of ρ_1 in comparison with ρ_0.

Let us carry out numerical estimates, choosing an aqueous solution of a dye [4] as the coolant, and assuming that α, β, and c_p are independent of the temperature. We have (see, for example [5, 6]) $c_p = 1$ cal/g·deg; $\rho = 1$ g/cm^3; $\varkappa = 0.6$ W/m·deg; $\alpha_0 = 2.1 \cdot 10^{-4}$ liter/deg; β_0 $4.9 \cdot 10^{-5}$ liter/atm.

The condition $|\rho_1| \ll \rho_0$ applies a constraint on the flash energy density $W \ll (\rho_0 c_{p0}/\alpha_0 K \eta)$, where η is the emission efficiency of the flash tube in the filtered part of the spectrum (usually $\eta \sim 0.1$).

Choosing $K = 100$ cm^{-1}, we obtain $W \ll 2 \cdot 10^2$ J/cm^2 and $T \ll 5 \cdot 10^{3}$°C

$$\frac{\alpha_0^2 T}{\rho_0 \beta_0 c_{p0}} \ll \frac{\alpha_0}{\rho_0 \beta_0 c_{p0}} \sim 0,\ 1.$$

In order to avoid boiling, the parameters of the system must be chosen so that the inequality

$$\frac{R_2^2 - R_1^2}{2R_1} K < \gamma_0 \frac{\alpha_0}{\beta_0} \cdot \frac{T_c - T_0}{p_c - p_0} \tag{11}$$

is satisfied.

In the case considered, condition (11) is satisfied when $R_2 - R_1 \sim 0.1$ cm ($R_1 \gg R_2 - R_1$). For this gap thickness and $W = 30$ J/cm^2, we obtain from formula (9) that $p - p_0 = 30$ atm. A commercial-type flash tube withstands this pressure without distortion [7]. Moreover, it exerts a compensating effect on the internal pressure in the flash tube. This should increase the limiting loads and the life of flash tubes in accordance with the results of [1].

We approximate the function $w(t)$ by the relation $w_0 \exp(-t/\tau)$ (w_0 = const). With this approximation, the temperature of the liquid at the start of the flash increases more rapidly than in actual cases, for which $w(0) = 0$. A typical value of τ for flash tubes is $5 \cdot 10^{-4}$ sec [8]. For this value of τ we obtain that the flow of energy in the liquid, due to thermal conductivity, is equal to the flow of radiant energy during $\sim 5\tau$. Therefore, it is permissible to neglect the thermal conductivity of the liquid in calculations, during the entire flash.

NOTATION

τ	is the duration of flash;
u	is the velocity of sound in the liquid;
R_1	is the external radius of flash-tube container;
R_2	is the internal radius of reflector;
T	is the temperature of liquid;
p	is the pressure of liquid;
ρ	is the density of liquid;
q	is the energy flux density;
S	is the Poynting's vector in the filtered range of frequencies;
c_V, c_p	are the specific heats;
ε	is the specific internal energy;
α	is the coefficient of volume expansion,
β	is the isothermal compressibility;
$\varkappa$	is the coefficient of thermal conductivity;
ρ_n	is the density of gas phase;
T_s	is the critical temperature of liquid;
p_c	is the critical pressure;
W	is the energy density of flash;
$\gamma \equiv c_p/c_V$.	

LITERATURE CITED

1. V. M. Podgaetskii, Zh. Priklad. Spektrosk., 20, No. 1, 48 (1974).
2. N. M. Kuznetsov, Zh. Prikl. Mekh. i Tekh. Fiz., 1, 112 (1961).
3. N. G. Byalko, G. A. Matyushin, and B. V. Tolkachev, Zh. Prikl. Spektrosk., 16, No. 5, 916 (1972).
4. V. M. Volynkin, A. A. Mak, Yu. N. Mikhailov, A. K. Pogodaev, and A. I. Stepanov, Opt.-Mekh. Prom-st', No. 3, 65 (1968).

5. N. B. Vargaftik, Handbook on the Thermophysical Properties of Gases and Liquids [in Russian], Nauka, Moscow (1972).
6. Handbook of Chemistry [in Russian], Khimiya, Leningrad (1971).
7. V. P. Kirsanov, Author's Abstract Candidate's Dissertation [in Russian], Moscow (1970).
8. B. P. Belostotskii, Yu. V. Lyubavskii, and V. M. Ovchinnikov, Principles of Laser Techniques [in Russian], Sovet-skoe Radio, Moscow (1972).